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The Resolution of the Optimal Reactive Power Dispatch with Continuous and Discrete Variables

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ABSTRACT

This paper presents a strategy for solving the optimal reactive dispatch (ORD) problem with continuous and discrete control variables; the tap ratios of transformers and shunt susceptances of switchable capacitor banks and reactors are the discrete control variables. The discrete control variables are treated as continuous and handled by either a sinusoidal or a polynomial discretisation function. This elaborate modelling of discrete variables results in a modified ORD and, due to the proposed resolution algorithm, it can be efficiently solved by nonlinear programming methods, such as the Generalised Reduced Gradient algorithm. The efficiency and robustness of the proposed modelling are shown for IEEE benchmark test-systems with up to 300 buses. Test results show the success in providing good solutions while attaining directly the discrete values of controls without further approximations.

KEYWORDS. Discrete variables. Nonlinear programming. Reactive optimal power flow.

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1. Introduction

Since its introduction in 1962 [Carpentier, 1962], the optimal power flow (OPF) has been considered as an important tool for planning, management and operation of electrical systems [Huneault and Galiana, 1991; Momoh et al., 1999a,b; Pandya and Joshi, 2008; Frank et al., 2012a,b]. The development of robust solvers for such a complex problem has been a timely research topic [Quintana et al., 2000], however, it still remains a challenge for general industry large-scale deployment.

One of its many applications is the definition of control actions that result in the minimization of active power losses in transmission systems. In this case, the control variables are the ones related to reactive power such as voltage magnitudes at generation buses, tap ratios of on-load tap changing (OLTC) transformers and shunt susceptances of switchable capacitor banks and reactors; this problem can also be referred to as the ORD problem, since all control variables associated with active power are fixed. Among all possible control adjustments, some discrete controls are, for the sake of simplicity, considered fixed [Oliveira et al., 2015; Pinheiro et al., 2015] or modelled as continuous variables [Capitanescu and Wehenkel, 2013].

When discrete control variables are considered continuous, fraction numbers are subsequently adjusted or "rounded off" to their nearest discrete value [Adibi et al., 2003]. In OPF problems, this approach may yield non-optimal or even result in unfeasible solutions [Liu et al., 1992; Soler et al., 2013]. Mixed integer nonlinear programming (MINLP) problems are frequently solved by mixed integer programming (MIP) solvers; however, the computational burden imposed by the combinatorial nature tends to increase exponentially with the number of discrete variables and even commercial solvers are not able to find good solutions for large size systems [Soler et al., 2013; Tofoli et al., 2018].

The resolution proposed in this paper goes further on exploring alternative methods of solving the ORD with continuous and discrete control variables by means of classical nonlinear programming (NLP) techniques; this research does not consider enumerative algorithms such as branch-and-bound or decomposition methods. The motivation is to explore the concept of applying efficient discretization functions such as in [Soler et al., 2013], where the authors proposed the use of a sinusoidal discretization function for handling discrete control variables. The original MINLP problem is modified as an equivalent NLP model without discrete variables.

In this paper, an enhancement is obtained with the proposal of a strategy that applies both the sinusoidal discretization function and a polynomial discretization function initially proposed by [Gisvold and Moe, 1972], and later modified by [Mazzini and Asada, 2014]. The objective is to explore the suitability of discretization functions that handle different controls with distinct impacts and degrees of adjustment in the ORD problem, such as tap ratios of OLTC transformers and shunt susceptances of switchable capacitor banks and reactors. The composite use of a sinusoidal function, wich provides best performance for regularly spaced discrete steps such as transformer tap ratios, and a polynomial function, applied for irregularly spaced discrete steps for switchable capacitor banks and reactors, is proposed.

The modified ORD model (with only continuous variables) is then solved by solver CONOPT, which applies the Generalised Reduced Gradient (GRG) method [Drud, 1985, 1992]. Tests have been carried out with IEEE test-systems of 14, 30, 57, 118 and 300 buses.

The remaining of this paper is organised as follows. In Section 2, the mathematical formulation of the ORD problem with continuous and discrete control variables is presented. In Section 3, the strategies for handling control variables adjusted by discrete steps are described. In Section 4 the GRG method used by the solver CONOPT is described. Section 5 outlines the numerical tests

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and main results. The main contributions of this paper are summarised and highlighted in Section 6.

2. The Optimal Reactive Dispatch with Continuous and Discrete Variables

The ORD problem can be formulated as follows:

$$
\begin{array}{lll}
\min & \sum_{k,m\,\in\,\mathcal{L}\cup\mathcal{T}} g_{km}\left(\frac{1}{t_{km}^2}V_k^2 + V_m^2 - 2\frac{1}{t_{km}}V_kV_m\cos\theta_{km}\right) & \\
\text{s.t.:} & & \\
& P_k - \sum\limits_{m\in\mathcal{V}_k} P_{km}(V,\theta,t) = 0 & \forall\, k \in \mathcal{G}'\cup\mathcal{C}, \\
& Q_k + Q_k^{sh}(V_k, b_k^{sh}) - \sum\limits_{m\in\mathcal{V}_k} Q_{km}(V,\theta,t) = 0 & \forall\, k \in \mathcal{C}, \\
& Q_{G_k}^{lo} \le Q_{G_k}(V,\theta,t,b_k^{sh}) \le Q_{G_k}^{up} & \forall\, k \in \mathcal{G}, \\
& V_k^{lo} \le V_k \le V_k^{up} & \forall\, k \in \mathcal{B}, \\
& b_k^{sh} \in \mathcal{D}_k^{sh} & \forall\, k \in \mathcal{B}^{sh}, \\
& t_{km} \in \mathcal{D}_{km}^{tap} & \forall\, k,m \in \mathcal{T},
\end{array}
\tag{1}
$$

V and θ are, respectively, the vectors of voltage magnitudes and phase angles on each bus; t is the vector of tap ratios of OLTC transformers; b^{sh} is the vector of shunt susceptances of switchable capacitor banks and reactors; P_k and Q_k are, respectively, the active and reactive power injections at bus k; Q_{G_k} is the reactive power generation at bus k; Q_k^{sh} is the reactive power injection at bus k by shunt elements; P_{km} and Q_{km} are the active and reactive power flows on each branch k-m; $\theta_{km} = \theta_k - \theta_m$; and g_{km} is the series conductance of branch k-m; $Q_{G_k}^{lo}$ and $Q_{G_k}^{up}$ are lower and upper bounds of Q_{G_k}, V_k^{lo} and V_k^{up} are lower and upper bounds of V_k.

In addition, $\mathcal{B}$ is the set of all system buses; $\mathcal{G}$ is the set of generation buses; $\mathcal{G}'$ is the set of generation buses except for the slack bus, $\mathcal{C}$ is the set of load buses; $\mathcal{B}^{sh}$ is the set of buses with voltage magnitude control by shunt capacitor banks and reactors; $\mathcal{L}$ is the set of branches k-m that represent transmission lines; $\mathcal{T}$ is the set of branches k-m that represent OLTC transformers, $\mathcal{V}_k$ is the set of bsses connected to bus k; and, at last, $\mathcal{D}_{km}^{tap}$ is the set of discrete values of transformer tap ratios and $\mathcal{D}_k^{sh}$ is the set of discrete values of shunt susceptances of switchable capacitor banks and reactors.

3. Strategies for Handling Discrete Variables

Consider the general NLP problem (2) with continuous and discrete variables:

$$
\begin{array}{lll}
\min & f(x) & \\
\text{s.t.:} & g_i(x) = 0 & i = 1,\cdots,p \\
& h_i(x) \le 0 & i = 1,\cdots,q \\
& x_{1_i}^{lo} \le x_{1_i} \le x_{1_i}^{up} & i = 1,\cdots,m_1 \\
& x_{2_i} \in \mathcal{D}_{x_{2_i}} & i = 1,\cdots,m_2
\end{array}
\tag{2}
$$

where $x_1 \in \mathbb{R}^{m_1}$ is the vector of continuous variables and $x_2 \in \mathcal{D}_{x_{2_i}} \subset \mathbb{R}^{m_2}$ is the vector of discrete variables, with $x = (x_1, x_2)$ and $x \in \mathbb{R}^n$; $\mathcal{D}_{x_{2_i}}$ is the set of discrete values for each variable x_2; $x_{1_i}^{lo}, x_{1_i}^{up} \in \mathbb{R}^{m_1}$ are the vectors of lower and upper bounds of the now continuous variables x_1; $f : \mathbb{R}^n \mapsto \mathbb{R}$; $g : \mathbb{R}^n \mapsto \mathbb{R}^p$, with $p < n$; and $h : \mathbb{R}^n \mapsto \mathbb{R}^q$.

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According to [Soler et al., 2013], (2) is modified into (3) and the discrete variable x_2 can be considered continuous.

$$\begin{array}{lll} \min & f(x) + \omega \sum_{i=1}^{m_2} \varphi(x_{2_i}) & \\ \text{s.t.:} & g_i(x) = 0 & i = 1, \cdots, p \\ & h_i(x) \leq 0 & i = 1, \cdots, q \\ & x_{1_i}^{lo} \leq x_{1_i} \leq x_{1_i}^{up} & i = 1, \cdots, m_1 \\ & x_{2_i}^{lo} \leq x_{2_i} \leq x_{2_i}^{up} & i = 1, \cdots, m_2 \end{array} \tag{3}$$

where ω is a weighting factor for function φ.

This modified problem (3) has f, g and h being $\mathcal{C}^2$ class functions, and $x_{2_i}^{lo}, x_{2_i}^{up} \in \mathbb{R}^{m_2}$ are the vectors of lower and upper bounds of the now continuous variables x_2, with $x_{2_i}^{lo} = \min\{\mathcal{D}_{x_{2_i}}\}$ and $x_{2_i}^{up} = \max\{\mathcal{D}_{x_{2_i}}\}$, for $i = 1, \cdots, m_2$.

Observe the term $\omega \sum_{i=1}^{m_2} \varphi(x_{2_i})$, incorporated onto the objective function in (3), assumes null value whenever x_{2_i} takes on a value in $\mathcal{D}_{x_{2_i}}$. The performance of nonlinear solvers is affected by the differentiability and continuity of the discretization function φ.

Therefore, two discretization functions will be considered for the formulation of the ORD problem. The first one is a sinusoidal function proposed by [Soler et al., 2013] and the second one is the polynomial function initially proposed by [Gisvold and Moe, 1972].

3.1. The Sinusoidal Discretization Function

The sinusoidal discretization function is well suited as a discretization function since it is continuous, differentiable and periodic. Its main advantages are shown in [Soler et al., 2013]. The sinusoidal discretization function $\phi : \mathbb{R} \mapsto \mathbb{R}$ is presented as follows:

$$\phi(y_i) = \left[\sin\left(\frac{y_i}{y_i^{sup} - y_i^{inf}} \pi + \alpha_i \right) \right]^2, \tag{4}$$

where y_i^{sup} is the nearest feasible upper discrete value of y_i; y_i^{inf} is the nearest feasible lower discrete value of y_i; and α_i is a constant defined within $[0; \pi]$ such that ϕ is null when $y_i = x_{2_i} \in \mathcal{D}_{x_{2_i}}$. Since $\sin(n_i \pi) = 0$ for any integer n_i, the constant α_i is calculated according to the following equation

$$\frac{x_{2_i}}{x_{2_i}^{sup} - x_{2_i}^{inf}} \pi + \alpha_i = n_i \pi \;\Rightarrow\; \alpha_i = \left(n_i - \frac{x_2}{x_{2_i}^{sup} - x_{2_i}^{inf}} \right) \pi. \tag{5}$$

Therefore, as $\alpha_i \in [0; \pi]$, n_i must be equal to the nearest upper integer to:

$$\frac{x_{2_i}}{x_{2_i}^{sup} - x_{2_i}^{inf}} \tag{6}$$

When the set of discrete values $\mathcal{D}_{x_2}$ has regularly spaced values, α is constant. When the set of discrete values $\mathcal{D}_{x_2}$ has irregularly spaced values, α must be specified for each interval of $\mathcal{D}_{x_2}$.

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3.2. The Polynomial Discretization Function

The polynomial discretization function was originally proposed by [Gisvold and Moe, 1972] to general MINLP, and presents good properties such as continuity and differentiability similarly to sinusoidal function, being the major difference the lack of periodicity. The definition of the polynomial discretization function is as follows.

Given $y \in \mathbb{R}^{m_2}$, a polynomial function $\psi : \mathbb{R} \mapsto \mathbb{R}$ is defined for the discrete variables x_2 to solve problems such as (2), as follows:

$$\psi(x_2) = \sum_{j \in \mathcal{D}} \left(4q_j(1 - q_j)\right)^2, \tag{7}$$

where

$$q_j = \frac{(x_{2_j} - x_{2_j}^{inf})}{(x_{2_j}^{sup} - x_{2_j}^{inf})} \tag{8}$$

where $x_{2_j}^{sup}$ is the nearest feasible upper discrete value of x_{2_j}, and $x_{2_j}^{inf}$ is the nearest feasible lower discrete value of x_{2_j}. The maximum value of each term in the summation of (7) is 1 and the function is continuous and smooth in its first derivative over discretization points.

3.3. Discretization Algorithm

In order to obtain discrete values without resorting to rounding off strategies, the proposed algorithm consists in solving a sequence of NLP problems (3) for crescent values of ω. The variable ω is increased linearly by the factor $\hbar$, which is set within $(1, 10]$. The successive resolution of (3) and the parameter setting of ω continues until one of two stopping criteria is met, resulting in the Algorithm 1 shown in the following. Algorithm 1 can be used for both sinusoidal and polynomial discretization functions.

Algorithm 1: Discretization Algorithm for ORD

Given (2), define the φ function for each variable x_2 and build (3);
Set $k = 0$;
Set $x_1^{(0)}$, $x_2^{(0)}$, $\omega^{(0)}$, $\hbar$, ξ_1, ξ_2;
while $x_2^{(k)}$ *does not satisfy* $\left\|x_2^{(k)} - x_2'\right\|_\infty \leq \xi_1$ *or* $\left\|x_2^{(k)} - x_2^{(k-1)}\right\|_\infty < \xi_2$ **do**
  Solve (3) for $\omega^{(k)}$;
  Save $(x_1^{(k)}, x_2^{(k)})$ as the current solution;
  $\omega^{(k+1)} = \hbar\omega^{(k)}$;
  $k = k + 1$;
end

3.3.1. Stopping criteria

Criterion 1 (C1): $\left\|x_2^{(k)} - x_2'\right\|_\infty \leq \xi_1$. This criterion tests the convergence of the algorithm to a solution of the original problem, and it evaluates the discretization of the variables x_2 in the current solution. $x_{2_i}' \in \mathcal{D}_{x_{2_i}}$ is the nearest discrete value of $x_{2_i}^k$, for $i = 1, \cdots, m_2$, ξ_1 is the discretization tolerance and k represents the current iteration.

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Criterion 2 (C2): $\left\|x_2^{(k)} - x_2^{(k-1)}\right\|_\infty < \xi_2$. The second criterion evaluates the updating of the variable x_2 between two consecutive iterations. If the largest update for variable x_2 is less than a tolerance ξ_2, the algorithm is interrupted to avoid an unlimited increment of ω and the divergence on the execution.

3.4. Mixed Strategy

The mixed discretization strategy explores the best features of both ϕ and ψ discretization functions presented earlier. The discrete variables are handled by a combination of sinusoidal and polynomial discretization functions.

Consider the following NLP problem with continuous and discrete variables:

$$\begin{array}{lll} \min & f(x) & \\ \text{s.a:} & g_i(x) = 0 & i = 1, \cdots, p \\ & h_i(x) \leq 0 & i = 1, \cdots, q \\ & x_{1_i}^{lo} \leq x_{1_i} \leq x_{1_i}^{up} & i = 1, \cdots, m_1 \\ & x_{2_i} \in \mathcal{D}_{x_{2_i}} & i = 1, \cdots, m_2 \\ & x_{3_i} \in \mathcal{D}_{x_{3_i}} & i = 1, \cdots, m_3 \end{array} \tag{9}$$

where $x = (x_1, x_2, x_3)$ and $x \in \mathbb{R}^n$; $f : \mathbb{R}^n \mapsto \mathbb{R}$; $g : \mathbb{R}^n \mapsto \mathbb{R}^p$, with $p < n$; and $h : \mathbb{R}^n \mapsto \mathbb{R}^q$, $x_1 \in \mathbb{R}^{m_1}$ is the vector of continuous variables of the problem; $x_2 \in \mathcal{D}_{x_2} \subset \mathbb{R}^{m_2}$ are the vector of discrete variables adjusted by regular discrete steps; $x_3 \in \mathcal{D}_{x_3} \subset \mathbb{R}^{m_3}$ represents the vector of discrete variables adjusted by non-constant discrete steps.

The sinusoidal discretization function is applied for regularly spaced discrete variables and the polynomial discretization function is used to handle irregularly spaced discrete variables. By incorporating the ϕ function (4) and the ψ function (7) into the objective function of (9), the following modified problem is obtained:

$$\begin{array}{lll} \min & f(x) + \gamma \sum_{i=1}^{m_2} \phi(x_{2_i}) + \nu \sum_{i=1}^{m_3} \psi(x_{3_i}) & \\ \text{s.a:} & g_i(x) = 0 & i = 1, \cdots, p \\ & h_i(x) \leq 0 & i = 1, \cdots, q \\ & x_{1_i}^{lo} \leq x_{1_i} \leq x_{1_i}^{up} & i = 1, \cdots, m_1 \\ & x_{2_i}^{lo} \leq x_{2_i} \leq x_{2_i}^{up} & i = 1, \cdots, m_2 \\ & x_{3_i}^{lo} \leq x_{3_i} \leq x_{3_i}^{up} & i = 1, \cdots, m_3 \end{array} \tag{10}$$

where $x = (x_1, x_2, x_3)$ and $x_1 \in \mathbb{R}^{m_1}$, $x_2 \in \mathbb{R}^{m_2}$ and $x_3 \in \mathbb{R}^{m_3}$ are vectors of continuous variables. Functions f, g and h are C^2 class functions; $x_2^{lo}, x_2^{up} \in \mathbb{R}^{m_2}$ are the vectors of lower and upper bounds of the continuous variables x_2, with $x_{2_i}^{lo} = \min\{\mathcal{D}_{x_{2_i}}\}$ and $x_{2_i}^{up} = \max\{\mathcal{D}_{x_{2_i}}\}$ for $i = 1, \cdots, m_2$; $x_3^{lo}, x_3^{up} \in \mathbb{R}^{m_3}$ are the vectors of lower and upper bounds of the continuous variables x_3, with $x_{3_i}^{lo} = \min\{\mathcal{D}_{x_{3_i}}\}$ and $x_{3_i}^{up} = \max\{\mathcal{D}_{x_{3_i}}\}$ for $i = 1, \cdots, m_3$.

4. Generalised Reduced Gradient

The GRG was proposed by [Wolf, 1967] to solve NLP problems with linear constraints. Subsequently, [Abadie and Carpentier, 1969] extended this method to the direct treatment of inequality constraints and nonlinear constraints resulting in the GRG method. The GRG method separates the set of variables into basic and non-basic variables and modifies the constraints so that the necessary change on the basic variables can be computed directly from non-basic variables

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Algorithm 2: Discretization Algorithm for ORD by Means of the Mixed Strategy

Given (9), define ϕ for each x_2, ψ for each x_3, and build (10);
Set $k = 0$;
Set $x_1^{(0)}$, $x_2^{(0)}$, $x_3^{(0)}$, $\gamma^{(0)}$, $\nu^{(0)}$, c, ℓ, ξ_1, ξ_2;
while $x_2^{(k)}$ *and* $x_3^{(k)}$ *do not satisfy C1 or C2 criteria* **do**
  Solve (10) for $\gamma^{(k)}$ and $\nu^{(k)}$;
  Save $(x_1^{(k)}, x_2^{(k)}, x_3^{(k)})$ as the current solution;
  $\gamma^{(k+1)} = c\gamma^{(k)}$;
  $\nu^{(k+1)} = \ell\nu^{(k)}$;
  $k = k + 1$;
end

which corresponding slack variables are introduced for all inequality constraints and nonlinear constraints are linearised in the current point of operation. Therefore, GRG is defined as the objective function that calculates the total incremental change in the objective function considering both basic and non-basic variables. The GRG algorithm implemented in CONOPT initially proposed by Abadie and Carpentier [1969] has been used for the resolution of the corresponding NLP problem with continuous and discrete variables as (9). Details of the algorithm can be found in [Drud, 1985] and [Drud, 1992].

5. Test Results

The test systems used to verify the robustness and efficacy of the presented approach are the IEEE benchmark test-systems with 14, 30, 57, 118 and 300 buses.

Table 1 shows the main characteristics of the ORD problem for each test-system.

Table 1: Characteristics of the ORD Problem for Each Test-System

Test System	Equality Constraints	Continuous Variables	Discrete Variables
14	22	27	4
30	53	59	6
57	106	113	18
118	181	235	23
300	530	599	121

The ORD model was implemented in General Algebraic Modeling System (GAMS) [GAMS, 2013]. For all systems, the minimum and maximum voltage magnitude at each bus were considered to be 0.90 p.u. and 1.10 p.u., tap ratios of OLTC transformers were considered to vary between 0.88 p.u. and 1.12 p.u. with steps of 0.0075 p.u. (33 positions), while steps of the shunt susceptances vary according to each system, for more information on the specifications used see Mazzini [2016]. In all tests, ξ_1 and ξ_2 tolerances were 10^{-4}.

Three levels of loading scenarios have been tested for each system: light, medium and heavy loading. In order to generate the different loading levels, the base case specified in data base was considered as the light loading. Then, the system load was increased until the voltage magnitude in one load bus reached its lower limit, determining the heavy loading. It is also important to

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mention that a distributed slack bus was considered in the generation increase, and all loads were increased with constant power factor.

Figures 1 and 2 shows the optimal tap ratios of OLTC for the IEEE 300-Bus Test-System under heavy loading for continuous transformers and discrete transformers, respectively. Figures 3 and 4 shows the optimal susceptance of shunt elements for the IEEE 300-Bus Test-System under heavy loading for continuous susceptance of shunt elements and discrete susceptance of shunt elements, respectively.

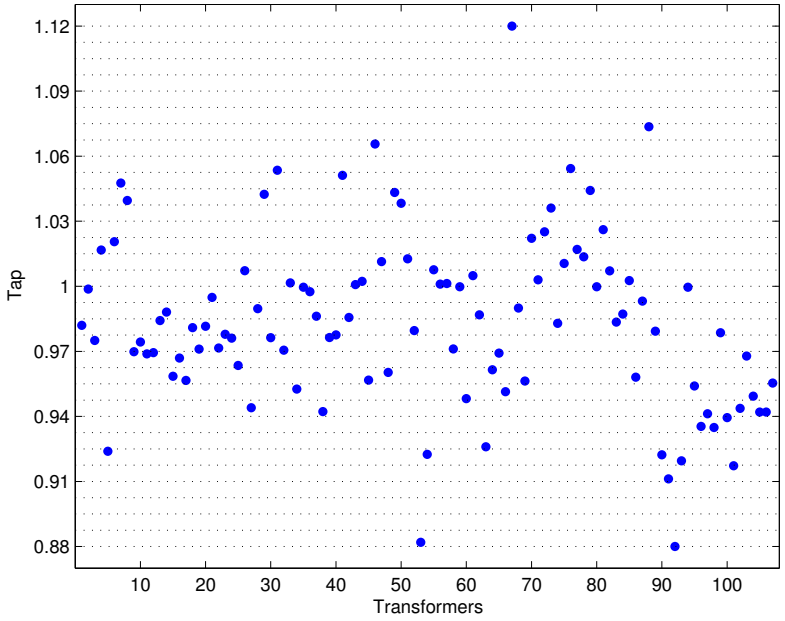


Figure 1: Optimal Continuous Tap Settings for the IEEE 300-Bus Test-System under Heavy Loading.

5.1. Analysis on the Results

Considering each system tested for all loading scenarios, we conclude for the IEEE-14, IEEE-30 and IEEE-118 systems, the application of different discretization strategies present similar solutions. The losses are almost the same for all approaches, with little difference from each other in the computing time and/or number of iterations. Moreover, in some cases, the same active losses are found for different local optima. One possible explanation is the low complexity to find optimal discrete solution for these systems; little can be mentioned about these results besides the fact the three discretization approaches are efficient in finding a good quality local optimal solution.

However, for IEEE-57 and IEEE-300 systems, the mixed strategy, considering both sinusoidal and polynomial discretization functions, is superior to the others regarding the computational time. The different system loading have been tested and a small impact on the number of iterations have been observed according to the increase of the system loading. Table 2 show the results obtained by employing the three approaches under system heavy loading.

It is possible to observe the solution enhancement regardless the presence of regular or not regularly spaced discrete variables. The choice of initial parameters of each algorithm is critical to the success of each approach. Overall, the mixed strategy presents at least the same or better performance than the others.

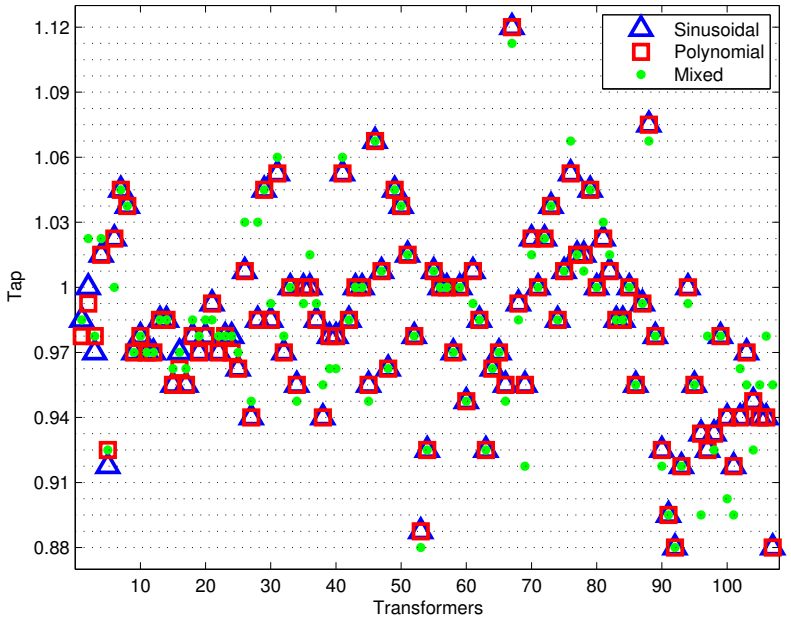


Figure 2: Optimal Discrete Tap Settings for the IEEE 300-Bus Test-System under Heavy Loading.

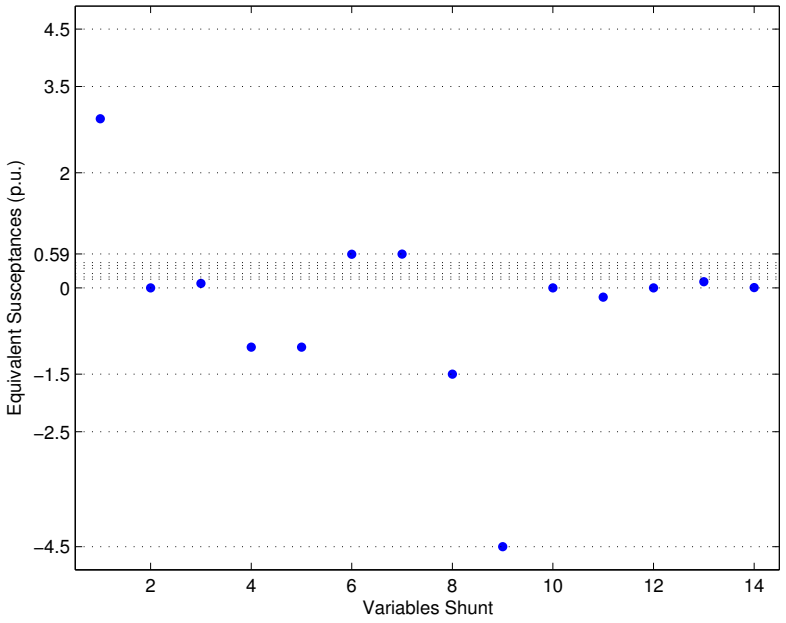


Figure 3: Optimal Continuous Shunt Susceptances for the IEEE 300-Bus Test-System under Heavy Loading.

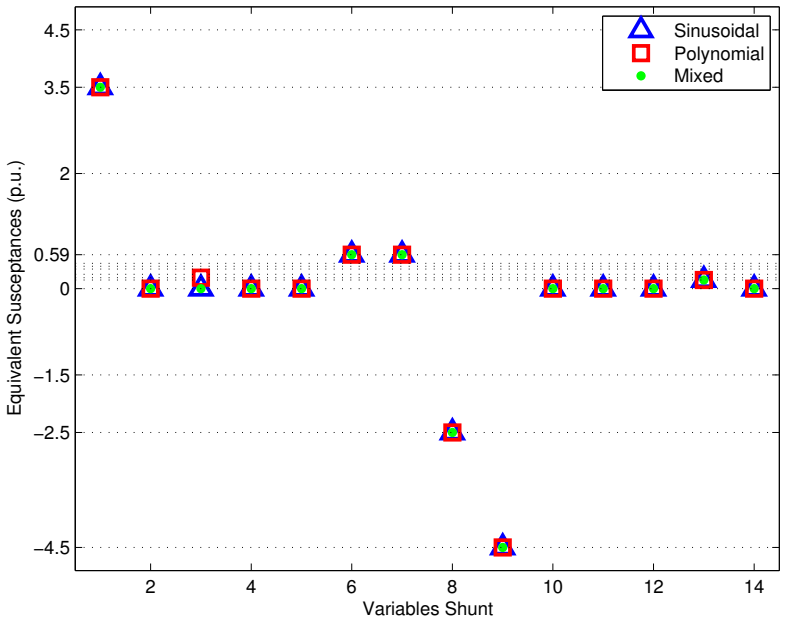


Figure 4: Optimal Discrete Shunt Susceptances for the IEEE 300-Bus Test-System under Heavy Loading.

6. Conclusion

The ORD is a NLP problem which nature is non-convex with continuous and discrete control variables. Its deployment for general application in the power industry still remains as a challenge. The method proposed in this paper explores the concept of deploying efficient discretisation functions for the corresponding discrete variables, which could then be represented by continuous variables. The developed approach used the sinusoidal function to handle the transformer tap ratios, which are adjusted by regular discrete steps, and the polynomial function to handle shunt susceptances of capacitor banks and reactors, which are adjusted by non-regular discrete steps.

Tests conducted with IEEE benchmark test-systems has shown good performance and efficiency to handle the discrete variables. For comparison purposes, tests have been conducted using the sinusoidal function and a polynomial function, for transformer tap ratios and shunt susceptances of capacitor banks and reactors. These results show the performance of the mixed strategy which can provide good solutions and has potential to overcome other strategies due to its tailored nature to model different discrete variables. Finally, the proposed strategy is general and has the potential to solve NLP problems with discrete variables in various fields of engineering.

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Table 2: Results for Test-Systems under **Heavy Loading**. $\gamma^{(0)}$, c, ν^0 and l are parameters of the discretization functions

Test System:	14 Bus	30 Bus	57 Bus	118 Bus	300 Bus
Continuous Relaxation					
Loss (MW)	30.78	27.55	25.09	261.98	371.65
Time (s)	0.140	0.234	0.313	0.891	3.297
Sinusoidal Approach					
$\gamma^{(0)}$	0.0001	0.00001	0.000001	0.000001	0.000001
c	10	10	10	10	10
Loss (MW)	30.78	27.56	26.70	262.01	374.60
Iterations	3	3	6	5	10
Time (s)	0.625	0.984	2.640	5.078	32.109
Polynomial Approach					
$\nu^{(0)}$	0.000953	0.000450	0.000465	0.000005	0.000630
ℓ	4.5	4.5	3.5	4.5	4.5
Loss (MW)	30.78	27.56	26.70	262.01	377.25
Iterations	2	2	5	6	10
Time (s)	0.469	0.875	2.703	5.578	45.734
Mixed Strategy					
Loss (MW)	30.78	27.56	26.67	262.01	373.00
Iterations	3	3	5	5	9
Time (s)	0.859	0.609	2.187	5.343	33.500

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