

BRAZILIAN HYDROTHERMAL DISPATCH OPTIMIZATION: REPLACEMENT OF INFLOW TREE BY LATTICE

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RESUMO

The Brazilian hydrothermal dispatch optimization has characteristics of a dynamic and stochastic problem. The stochasticity is represented by inflows tree, which increases exponentially with the raise of the number of stages analyzed, making harder the long-term planning. In this context, this article proposes the replacement of the representation of uncertainty scenarios from tree to lattice in order to reduce the computational effort to solve this problem. Lattices increase linearly and they are used when the scenarios are path independent. As a way to test the viability of this replacement it is presented a case study based in Tocantins basin data using tree and lattice. Comparing the results of the two methods it is possible to conclude that they are very close, so it is perchance possible to replace the inflow tree by lattice for longer periods when it would not be possible to apply the conventional method.

PALAVRAS CHAVE. Hydrothermal Dispatch, Stochastic Optimization, Lattice.

EN - OR in Energy, Oil and Gas Field

1. Introduction

A hydrothermal system of electricity generation consists predominantly of hydroelectric and thermoelectric plants, which are connected to the load centers through a transmission system. Hydroelectric plants generate energy through the water from rains, therefore have an almost null marginal cost of operation; while thermoelectric plants generate energy by burning fuel, having its marginal operating costs varying according to the price of inputs.

The basic objective of the operation planning of hydrothermal power system is to obtain, for each stage, the generation targets of each plant (hydro and thermal) of the system in order to meet the demand and minimize the expected value of operating costs over the planning period. To do so, it is needed to take into consideration that the optimization system has characteristics of a dynamic, interconnected, stochastic, nonlinear and large problem. The problem is dynamic, once the availability of water stored in the reservoir at a given time depends on the degree of its previous use; interconnected, since the hydroelectric power plants located in the same hydraulic basin have operating engagement with each other; stochastic, due to the uncertainty about future inflows and energy demand; non-linear, because of the cost of generation of thermoelectric power plants; and large due to the number of plants and restrictions that have to be considered in the resolution.

Among the models used in Brazil to perform the operation planning of the hydrothermal power system, the optimal allocation of electricity generation in the short-term horizon is achieved by DECOMP model [CEPEL, 2012a], and for the long-term horizon is utilized NEWAVE model [CEPEL, 2012b]. The stochasticity of the inflow is introduced in the models at the end of the first period of study by building inflows scenarios to the hydroelectrics. These hydrological scenarios can be represented by an inflow tree as the one exemplifying in Figure 1.

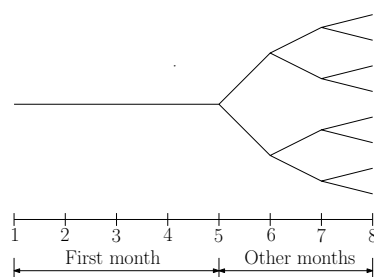


Figura 1: Scheme of a binomial tree utilized in DECOMP model.

Trees are used when the scenarios are path dependent. The number of paths (scenarios) of a tree is equivalent to the number of leaves (terminal nodes) and they increase exponentially with the raise of the number of stages analyzed, as can be seen in Table 1, which considers the stochasticity present from the first stage.

Tabela 1: Number of nodes in each stage of the tree.

number of initial scenarios	stage number				
	1	2	3	4	n
2	2	4	8	16	2^n
3	3	9	27	81	3^n
4	4	16	64	256	4^n
N	N	N^2	N^3	N^4	N^n

This exponential growth difficult to analyze the problem for long periods due to the large computational effort required. To get around this, NEWAVE and DECOMP models use Stochastic Dual Dynamic Programming (SDDP) technique [Pereira e Pinto, 1991] to subdivide the problem, making it still hard but feasible to solve computationally in the long term.

In this context, this article proposes the replacement of inflows tree by lattice in a way to reduce the computational effort to solve this problem. Lattices are used when the scenarios are path independent and they have the structure as shown in Figure 2. The number of terminal nodes of a lattice increases linearly while it still maintains the exponential growth of paths, as it is shown in Table 2. In this way, the lattice structure can represent the same number of scenarios of a tree with a more compact structure.

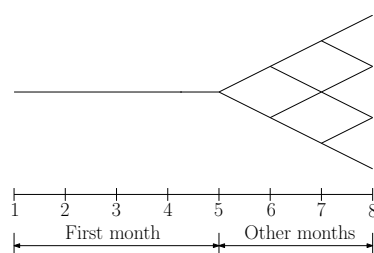


Figura 2: Scheme of a binomial lattice.

Tabela 2: Number of terminal nodes in each stage of lattice

number of initial scenarios	stage number				
	1	2	3	4	n
2	2	3	4	5	$n + 1$
3	3	5	7	9	$2n + 1$
4	4	7	10	13	$3n + 1$
N	N	$2N - 1$	$3N - 2$	$4N - 3$	$(N - 1)n + 1$

2. Literature Review

The theory of tree and lattice can be seen with more details in the book Investment Science [Luenberger, 1998]. The next subsections will present the adapted theory for our problem.

2.1. Tree

To solve the complete problem of the operation planning, a problem for each node of the inflow tree must be solved. Each node represents a decision that should be taken in each stage, and each decision leads through its branches to different results.

The inflow tree is assembled in the way that each node represents the volume of water kept in the reservoir and each branch represents a possible inflow scenario. Thus, for each branch there is a value associated with the generation, and for each node, there is a value of expected cost of generation as it is shown in the scheme of the Figure 3.

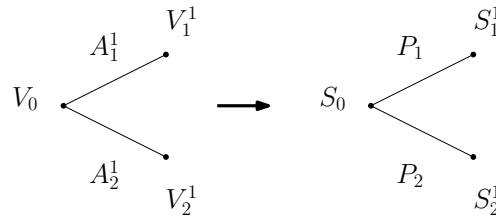


Figura 3: Scheme of representation of a tree.

The volume of water represented at each node is calculated by forward steps through the water balance equation, considering that the volume of the reservoir V_j^t at the end of a stage t in the node j is equivalent to the volume V_i^{t-1} in the node i at the stage $(t - 1)$, added to the inflow A_{ij}^t related to the branch ij connecting these nodes, discounted the amount of water used in hydroelectric generation HG_{ij}^t and the spillage I_{ij}^t in this period as shown in Equation (1).

$$V_j^t = V_i^{t-1} + A_{ij}^t - HG_{ij}^t - I_{ij}^t \quad (1)$$

Exemplifying for a simpler case, such as that of the binomial tree shown in Figure 4, Equations (2) to (7) are obtained.

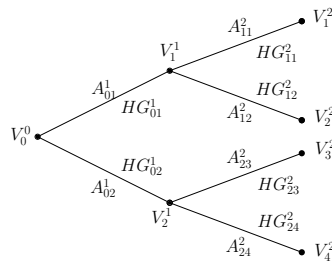


Figura 4: Scheme of calculation of water balance in binomial tree

$$V_1^1 = V_0^0 + A_{01}^1 - HG_{01}^1 - I_{01}^1 \quad (2)$$

$$V_2^1 = V_0^0 + A_{02}^1 - HG_{02}^1 - I_{02}^1 \quad (3)$$

$$V_1^2 = V_1^1 + A_{11}^2 - HG_{11}^2 - I_{11}^2 \quad (4)$$

$$V_2^2 = V_1^1 + A_{12}^2 - GH_{12}^2 - I_{12}^2 \quad (5)$$

$$V_3^2 = V_2^1 + A_{23}^2 - HG_{23}^2 - I_{23}^2 \quad (6)$$

$$V_4^2 = V_2^1 + A_{24}^2 - HG_{24}^2 - I_{24}^2 \quad (7)$$

Having calculated the hydroelectric generation of each stage, there is a cost of operation associated to each branch due to the generation by thermoelectric plants to complement the energy to meet the demand. Thus, it is possible to calculate the expected value of the cost of operation S_i^t of the stage t and node i by backward steps; and it is equivalent to the hope (or weighted average, according to the probability P_j of occurrence of the scenarios j) of expected values for the next stage. The Equation (8) shows this calculations, where N is the number of initial branches.

$$S_i^t = P_1 \cdot S_{(i-1) \cdot N+1}^{t+1} + P_2 \cdot S_{(i-1) \cdot N+2}^{t+1} + \dots + P_N \cdot S_{i \cdot N}^{t+1} \quad (8)$$

Exemplifying for the binomial tree shown in the Figure 5, whose branches present probability P and $(1 - P)$, the calculation of the expected value of each node is shown in Equations (9) to (11).

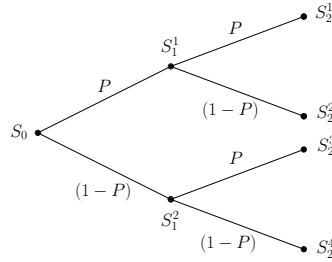


Figura 5: Scheme of calculation of the expected value of generation cost in binomial tree.

$$S_1^1 = P \cdot S_2^1 + (1 - P) \cdot S_2^2 \quad (9)$$

$$S_1^2 = P \cdot S_2^3 + (1 - P) \cdot S_2^4 \quad (10)$$

$$S_0 = P \cdot S_1^1 + (1 - P) \cdot S_1^2 \quad (11)$$

2.2. Lattice

The representation of the inflow scenarios by lattice follows the same principles of tree representation, however it is necessary to "force" the intermediate nodes to coincide, in an way that the values assumed by them are still continuous. In this way, calculations for the water balance and expected value of the operating cost would be smaller in quantitative terms, and are shown next.

Exemplifying for a simple case, such as that of the binomial lattice shown in Figure 6, Equations (12) to (17) are obtained using Equation (1).

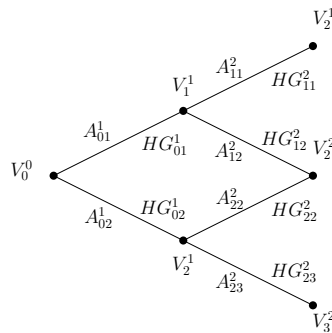


Figura 6: Scheme of calculation of water balance in binomial lattice

$$V_1^1 = V_0^0 + A_{01}^1 - HG_{01}^1 - I_{01}^1 \quad (12)$$

$$V_2^1 = V_0^0 + A_{02}^1 - HG_{02}^1 - I_{02}^1 \quad (13)$$

$$V_1^2 = V_1^1 + A_{11}^2 - HG_{11}^2 - I_{11}^2 \quad (14)$$

$$V_2^2 = V_1^1 + A_{12}^2 - HG_{12}^2 - I_{12}^2 \quad (15)$$

$$V_2^2 = V_2^1 + A_{22}^2 - HG_{22}^2 - I_{22}^2 \quad (16)$$

$$V_3^2 = V_2^1 + A_{23}^2 - HG_{23}^2 - I_{23}^2 \quad (17)$$

It is important to draw attention to calculations involving intermediate nodes, such as Equations (15) and (16), which must coincide, since they represent the volume calculations of the same node, the reason for to obtain two equations is due to the fact that it is possible to reach the same node through different paths.

The expected value S_i^t of node i at stage (t) is calculated using a analogous reasoning to that used by inflows trees and it is exemplify by Equations (18), (19) and (20) relating to lattice shown in Figure 7.

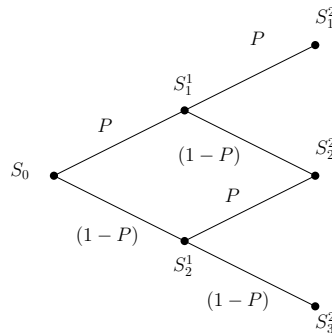


Figura 7: Scheme of calculation of the expected value of generation cost in binomial lattice.

$$S_1^1 = P \cdot S_1^2 + (1 - P) \cdot S_2^2 \quad (18)$$

$$S_2^1 = P \cdot S_2^2 + (1 - P) \cdot S_3^2 \quad (19)$$

$$S_0^0 = P \cdot S_1^1 + (1 - P) \cdot S_2^1 \quad (20)$$

In this case, the difference between lattice calculations and tree is the fact that the value S_2^2 is considered twice, since it is possible to arrive at this node through two different paths and, in this way, the intermediate node presents a higher probability of occurrence than the other final nodes.

It can be observed that the equations, both in the water balance and in the calculation of the expected value of the operating cost, are very similar to the equations used for the binary tree, however, there is a smaller number of end nodes, which leads to a smaller number of equations to be calculated if one more stage were added to the analyzed period.

3. Case study

3.1. Data

The data considered in this problem were obtained from the ONS (Operador Nacional do Sistema) website [ONS] and the CCEE (Câmara de Comércio de Energia Elétrica) website [CCEE]. And they were converted for the unit MWmed considering the time base of one month to avoid units mismatch problems during the optimization.

For the case study, we considered only the hydropowers in the Tocantins basin, shown in Figure 8. This plants were added in an equivalent hydroelectric plant (plant 1) in order to reduce the size of the problem, getting as maximum generation restriction the value of 12821.65 MWmed. The volume of the equivalent reservoir, was obtained through the ONS website, which shows that for the month of April 2015, the north subsystem reservoirs had a volume equivalent to 12006 MWmed corresponding to 81.06% of the total volume. Thus, the equivalent reservoir presents a restriction of the maximum volume corresponding to 14811.3 MWmed. The initial volume considered in the reservoir in the first stage of the problem was 10000 MWmed and after we changed it to 9000 MWmed; and the final volume should not be less than 4000 MWmed.

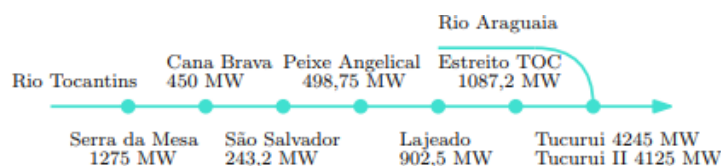


Figura 8: Hydroelectric plants in the Tocantins basin and their maximum capacity of generation ONS.

In addition, it were considered only the thermoelectric plants near the basin, its maximum generations and costs were obtained through the auctions consolidation table, available on the CCEE website, this data is shown in Table 3, where the thermoelectric Geramar I and Geramar II were considered as a single plant. And each power plant correspond to plants 2, 3 and 4, respectively, of the optimization program.

Tabela 3: Generation cost of thermoelectric power plants.

Power Plant	Power Generation(MW)	Type of Fuel	Price (R\$/MWh)
Maranhão III	499.2	Gas	127.04
Termomaranhão	350.0	Coal	198.60
Geramar I & II	330.0	Oil	211.04
Exchange energy	700.0	—	300.00

To calculate the demand to be attended, it was considered the north subsystem demand and its exchanges with the northeast subsystem between the years 2000 and 2014 (data available on the ONS operation history). The average score for each month is shown in Table 4.

Tabela 4: Demand data.

Month	North Subsystem (MWh/h)	N-NE Exchange (MWmed)	Total (MWmed)
January	3763.9	3905.7	7669.5
February	3813.6	3961.2	7774.8
March	3846.5	3996.7	7843.2
April	3890.4	4046.8	7937.2
May	3892.9	4044.1	7937.0
June	3885.9	4037.5	7923.4
July	3885.8	4061.0	7946.8
August	3983.0	4162.9	8145.9
September	3999.0	4193.6	8192.6
October	3973.4	4165.3	8138.7
November	3970.0	4162.9	8132.8
December	3989.9	4179.7	8169.7

And finally, for the insertion of the stochasticity in the problem, it were considered the Affluent Natural Energy history data between the years 1931 to 2013 for the Tocantins basin (available on ONS). From these data, it were made for each month the cumulative distribution of probability, and were caught average values for an optimistic scenario and a pessimistic scenario of inflow, so that each scenario has a 50% chance to occur. These values are shown in Table 5.

Tabela 5: Inflow Scenarios.

Month	Pessimistic Scenario (MWmed)	Average Inflow (MWmed)	Optimistic Scenario (MWmed)
January	10495.7	12765.0	16676.3
February	13416.4	16672.3	21907.0
March	15528.1	18511.7	25145.1
April	14313.3	17254.7	21255.8
May	8526.0	10676.1	13080.9
June	4534.5	5265.8	6598.0
July	2934.7	3404.5	4000.5
August	2118.3	2463.2	2885.7
September	1732.2	2089.4	2509.9
October	2027.2	2639.2	3335.3
November	3534.1	4197.5	5722.1
December	6304.1	8028.6	10643.0

As the number of scenarios increases according to the number of analyzed stages, the analysis presented in this article will only cover four stages (months of May, June, July and August, respectively). The choice of the months was based on the transition from high inflows to critical inflows.

Using these data, the resolution will be made through the traditional method that uses the inflow tree, and then by the proposed method using lattice. After that, the results will be compared to verify if the change is viable.

3.2. Results

To solve this problem it is necessary to define the objective function as the minimum expected value of the cost of the stages, subject to constraints equations of the problem (water balance equation, demand attendance and power restrictions). So the problem is solved applying the Stochastic Dual Dynamic Programming, that uses Benders Decomposition Method.

Figure 9 shows the inflow tree to be analyzed, the first stage is deterministic and each rectangle represents a sub problem to be solved by the SDDP method. For the resolution by binomial lattice, the representation of the sub problems to be solved is shown in Figure 10.

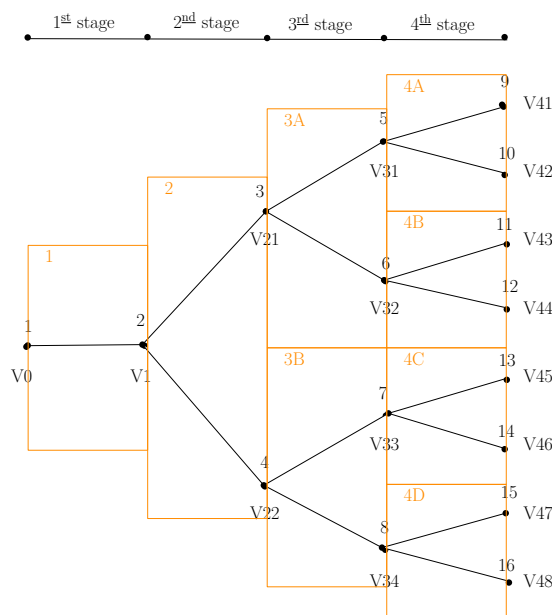


Figura 9: Layout of the inflow tree used in the problem.

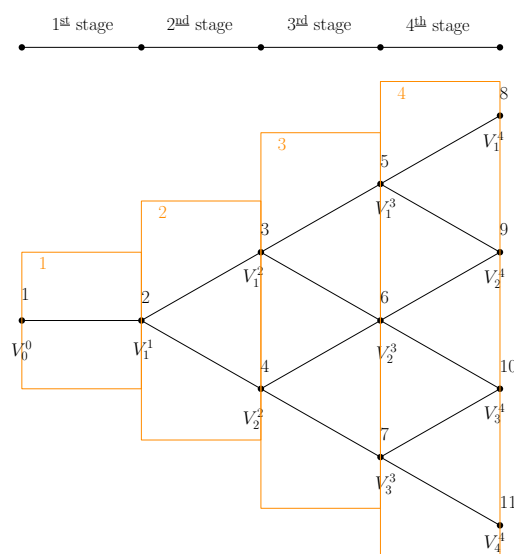


Figura 10: Layout of the lattice used in the problem.

The results obtained for the first case, in which the initial volume of the reservoir considered was 10000 MWmed, are shown in Figures 11 and 12. The goal of the objective function coincided, using binary tree or binomial lattice, and is R\$ 638781,20.

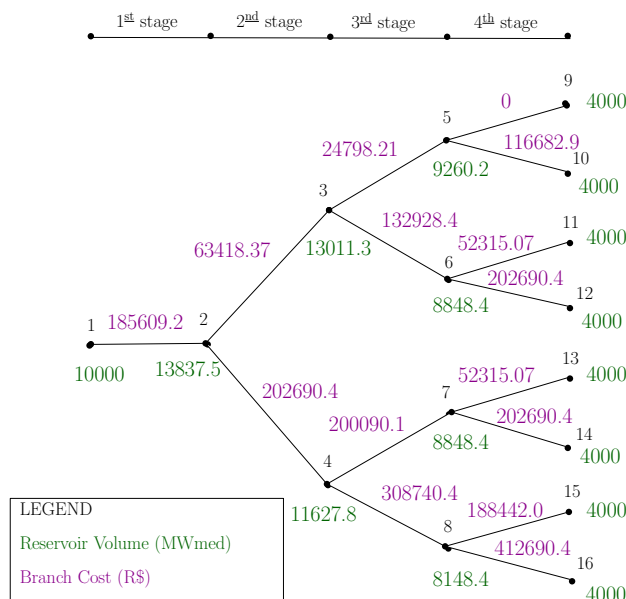


Figura 11: Solution using tree with 10000 MWmed as initial volume.

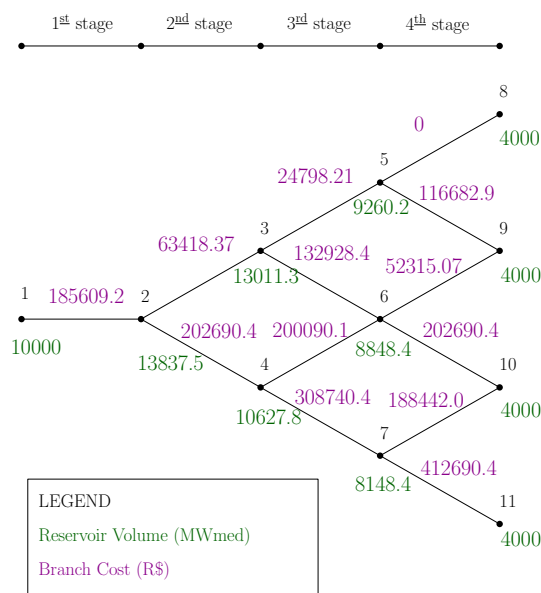


Figura 12: Solution using lattice with 10000 MWmed as initial volume.

Now, for the second case, where the initial volume of the reservoir considered was 9000 MWmed, the results are shown in Figures 13 and 14.

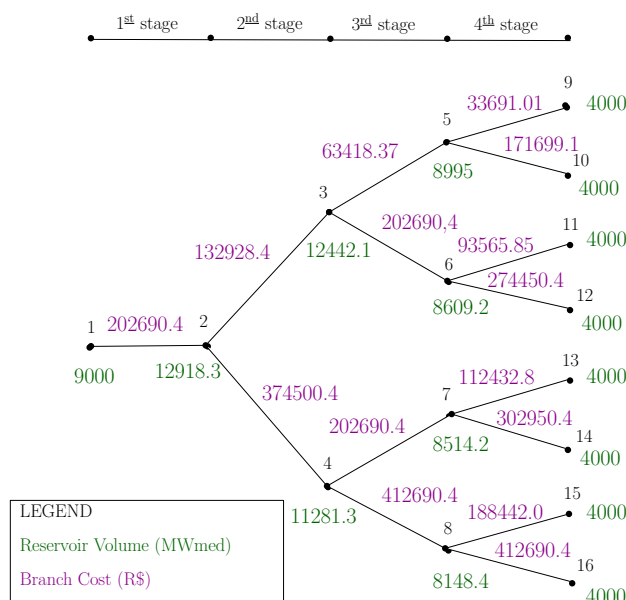


Figura 13: Solution using tree with 9000 MWmed as initial volume.

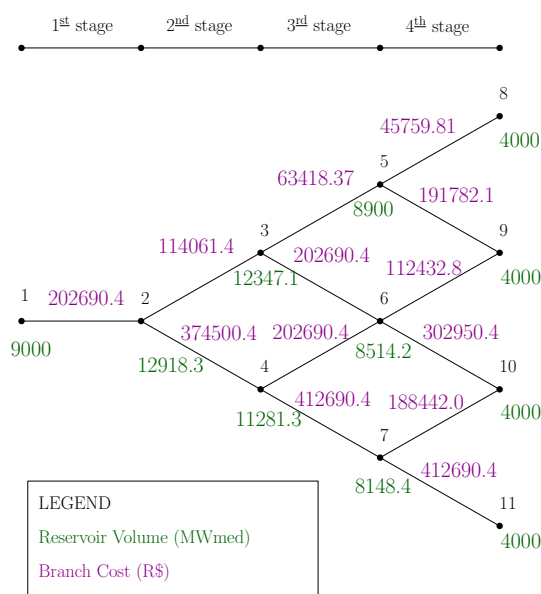


Figura 14: Solution using lattice with 9000 MWmed as initial volume.

As can be seen, the values obtained did not coincide. For resolution using the binary tree, the goal of the objective function of the expected cost of the operation was R\$ 875517,30. While for resolution through binomial lattice, the amount obtained from the expected cost of the operation was R\$ 876023,70.

In this way, it was verified that there may be cases in which the results obtained may be

different. However, it seems that even the results do not match, this one is very close.

Thus, through the case study presented here, the substitution of the lattice evidences was apparently feasible, even if there is no complete coincidence of the values presented.

The solution with details of an analogously problem using tree can be seen on the presentation "Representação de Aversão a Risco no Planejamento da Operação Hidrotérmica" [CCEE, 2012].

4. Conclusion

As we can see in the section results, the values obtained in the optimization of hydrothermal dispatch in the Tocantins basin using the two methods of resolution agreed in the first case and they were close in the second case.

The differences in the results in the second case could be occurred due to the fact that lattices are used when the scenarios are path independent. In the first case we have more water available in the initial stage of our problem and in the second case we have less water, the amount of water in the reservoirs could influence if the problem is path dependent or path independent. Since Brazil has hydroelectrics with large capacity for storing water in its reservoirs it is indifferent if in a month it rains a lot and in the other it rains a little or the reverse. So it is believed that it is possible to replace the inflow tree by a lattice in the Brazilian hydrothermal dispatch optimization.

The reservoirs with large storage capacity enables to have a greater control of the decision of how much the hydroelectrics will generate in each stage. Thus, the decisions of each node can walk through different branches in order to "tie" the intermediate nodes of the inflow tree, causing it to have the structure of a lattice.

Futhermore, this replacement could be possible because the problem of hydrothermal dispatch optimization modeled on DECOMP is considered as "chance-decision" type [CEPEL, 2012a], that is, knowing in advance the inflow occurred, the decision of the hydrothermal dispatch is taken so that the intermediate nodes coincide.

Besides that, the resolution using lattice requires less computational effort than the solution using inflow tree, since the growth of the structure is smaller. Thus, the use of less computational effort should compensates the sacrificing of the optimal solution.

In this way, it could be advantageous to apply this proposed replacement on the current models of hydrothermal dispatch optimization.

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